K_{e2} branching ratio $\leq 1.6 \times 10^{-3}$ with 95% confidence, which is still above the *V-A* theory prediction * (2.6×10^{-5}).

We thank Dr. R. Cester for suggesting the method, the Turin bubble chamber group for valuable help in the first part of this work and the staff of the CERN-Saclay 81 cm hydrogen chamber for the exposure.

1) A. Pais and S. B. Treiman, Phys. Rev. 105 (1957) 1616;
S. Furuichi, T. Kodama, S. Ogawa, Y. Sugahara, A. Wakasa and M. Yonezawa, Progr. Theoret. Phys. (Kyoto) 17 (1957) 89.
2) J. L. Brown, J. A. Kadyk, G. H. Trilling, R. T. Van de Walle, B. P. Roe and D. Sinclair, Phys. Rev. Letters 7 (1961) 423, 8 (1962) 450.

* See for example Jackson [9].

3) D. Luers, I. S. Mittra, W. J. Willis and S. S. Yamamoto, Phys. Rev. Letters 7 (1961) 255 and Phys. Rev. 133B (1964) 1276;
G. Kalmus, A. Kernan, U. Camerini and C. Henderson, 1964 Int. Conf. on High Energy Physics, Dubna, USSR.
4) Feynmann and Gell-Mann, Phys. Rev. 109 (1958) 193;
S. Weinberg, R. E. Marshak, S. Okubo, E. C. G. Sundarshan and W. B. Teutsch, Phys. Rev. Letters 1 (1958) 25.
5) J. Bernstein and S. Weinberg, Phys. Rev. Letters 5 (1960) 481;
H. Chew, Phys. Rev. Letters 8 (1962) 297.
6) S. W. MacDowell, Phys. Rev. 116 (1959) 1047;
P. Dennery and H. Primakoff, Phys. Rev. 131 (1963) 1334.
7) N. Brene, L. Egardt and B. Quist, Nucl. Phys. 22 (1961) 553.
8) B. Rossi, High energy particles (Prentice Hall, Englewood Cliffs, N. J., 1956) p. 15.
9) J. D. Jackson, 1962 Brandeis lectures, Volume I (W. A. Benjamin, New York, 1963).

* * * * *

MOTION OF SWIFT CHARGED PARTICLES, AS INFLUENCED BY STRINGS OF ATOMS IN CRYSTALS

J. LINDHARD
Institute of Physics, University of Aarhus, Aarhus, Denmark

Received 26 August 1964

It seems possible to formulate a comprehensive theory of directional effects for energetic charged particles in crystal lattices. The theoretical treatment leads to rather definite concepts, and makes desirable certain lines of experimental investigation. The detailed theory will be published shortly, and the present letter merely indicates a few basic ideas, for the case of swift particles, as well as theoretical reasons for attempting the new type of experiments presented in the following letter by Bøgh, Davies and Nielsen.

The problem of the path of a particle moving through a crystal lattice is primarily connected with its deflections, due to the screened Coulomb interaction between the particle and an atom as a whole. In such collisions it must pass close to or through the atom. The first circumstance to be noted is that for swift particles we need consider only small deflections in the laboratory system as well as in the centre of gravity system. When this is the case, deflections of the particle are determined by the potential of the crystal lattice, as if the atoms were infinitely heavy. Secondly, directional effects must be due to some kind of correlation between successive collisions with atoms in the crystal lattice. If a particle moves nearly along a major direction in a lattice and passes close to one atom, it must also pass close to the neighbouring atoms in the same row. From this result, one is led to the simple concept of a string of atoms, characterized only by a constant distance of separation, d, between atoms placed on a straight line. In first approximation, collisions occur successively with independent strings of atoms. Thirdly, most of the physical processes caused by the particle demand that it passes close to the atomic nucleus, or to atomic electrons. This shows that an insight in the collisions between particles and a string of atoms gives basic information about the occurrence of physical processes.

The following discussion is based on classical orbital pictures, both for localizing the particles in the lattice and in describing collisions with atoms. The classical approximation is favoured by the collective, smoothing effect of successive collissions with atoms in a string. For the same reason, vibrations of atomic strings may be disregarded at first, if the amplitudes are not excessively large.

Let ψ be the angle between particle motion and string direction, when the particle is far away from a string. There is a characteristic angle ψ_1 separating between particles with large ψ's entering deeply into strings and particles with small ψ's being always repelled in more glancing collisions. For swift particles, ψ_1 is given by

$$\psi_1 = \left(\frac{2Z_1Z_2e^2}{dE}\right)^{\frac{1}{2}} , \tag{1}$$

where Z_1 and Z_2 are the charge numbers of particle and atom, respectively, and E is the particle energy; d is the distance between atoms in the string. One also finds that in order to be able to reach the centre of atoms in a perfect string without vibrations, the particle must have an angle ψ exceeding $C\psi_1$, where $C \sim 1.5 - 2$. A lower limit to $C\psi_1$ may be obtained from a crude, but instructive model, where a beam of particles is scattered classically, according to the Rutherford scattering law, by one atom of charge, Z_2e. In this case, a parabolic shadow region is found behind the atom, such that the angle from beam direction to shadow edge is $\sqrt{2}\psi_1$, at a distance d behind the atom.

A useful approximation is to describe the string as a continuum, i.e., to replace the periodic string potential by the average potential at a distance r from the string. For a continuum string the angle ψ_1 may be obtained as follows. A screened atomic potential varies approximately as $Z_1Z_2e^2/R$, at distances $R < a$ from the atom, where a is the familiar Thomas-Fermi screening radius $a = 0.8853\ a_0Z_2^{-\frac{1}{3}}$ (this formula for a is applicable if $Z_1 \ll Z_2$, or if the particle velocity is high). At larger distances it falls off more quickly, e.g. as $Z_1Z_2e^2a/(2R^2)$ for $R > a$. Averaging the atomic potential along a string, we obtain a string potential $Z_1Z_2e^2\pi a/(2dr)$, if the distance r from the string axis is $r > a$, and only a slow logarithmic increase in the potential when $r < a$. We thus find a height of the potential $\sim 2Z_1Z_2e^2/d$, and are led to a formula of type of (1) for the characteristic angle, since for small angles the energy of the motion perpendicular to a string is $E\psi^2$. The orbits of particles colliding with a continuum string may also be derived. If we then demand that the continuum picture is consistent, i.e., that many atoms contribute to the orbital deflections, it can be seen that this again leads to $\psi \lesssim \psi_1$.

A beam of particles is now, according to (1), divided naturally in two, a random beam with angles $\psi \gtrsim C\psi_1$, and an aligned beam with $\psi \lesssim C\psi_1$. The particles of the random beam experience the lattice essentially as a random system, and suffer multiple scattering of normal kind. The random beam thus diffuses rapidly in a normal manner towards wider angles, and the path length λ_n on which its average square angle, $\overline{\psi^2}$, increases by $\psi_1{}^2$, corresponds approximately to an energy loss of order of $\delta E_1 \sim Z_1(M_1/M_p) \times 10$ keV, where M_p is the proton mass. The aligned beam does not come close to the centre of atoms, the less so the smaller the angle ψ, and therefore its multiple scattering decreases rapidly with ψ, and is partly due to thermal vibrations and crystal imperfections. However, for an aligned particle with angle ψ, there is a quick diffusion in azimuthal angle at fixed angle ψ with the strings. From scattering in collisions with perfect strings, the distribution in azimuthal angle is estimated to smooth out on a path length $\lambda \approx 2\psi/(Nad\psi_1{}^2)$, i.e., of order of λ_n. The above distinction between random and aligned beam is not quite sharp; in an intermediate region, between $\sim\psi_1$ and $\sim C\psi_1$, multiple scattering remains large.

Initially, if a beam hits the surface of a crystal at zero angle with strings, there is a minimum fraction of particles in the random beam. In fact, particles aiming at the end of an atomic string, within a distance $\sim a$ from its centre, will be deflected so as to pass rather quickly into the random beam, while the rest starts in the aligned beam, or at its edges. The minimum fraction of particles in the random beam is thus

$$\chi_{\min} \sim \pi a^2 dN , \tag{2}$$

where N is the number of atoms per unit volume. The formula (2) is merely a rough estimate.

The characteristics of the string effect are, firstly, the division into aligned and random beams. Secondly, there is confinement of aligned particle orbits to certain parts of space, mainly outside strings. Thirdly, there is a change in multiple scattering, i.e., in the diffusion of the direction of motion of the particles, such that the aligned beam has a slow change of angle ψ. Fourthly, the slowing-down of the two beams is different, the random beam having approximately normal stopping, whereas particles in the aligned beam have reduced electronic stopping, the more so the smaller the angle ψ. For fast particles, where the Bethe-Bloch stopping formula applies, the reduction in stopping is at most by a factor of ~ 2, since half of the energy loss is in close collisions, and half of it in distant resonance collisions.

These cursory considerations indicate also some promising lines of experimental investigation. Thus, the absence of penetration to the centre of strings may be measured by means of any process demanding that the particle comes very much closer than a to the atomic nucleus. It may be natural to attempt observations of the extinction of nuclear reactions, as reported in the following letter for (p,γ)-reactions. In heavy elements, measurements of Coulomb excitation are favourable; these processes also have the advantage that particles with different atomic numbers Z_1 may be used. For most purposes, observations of single Rutherford scattering at large angles have conspicuous advantages; the cross sections are large, and the energy, as well as Z_1 and Z_2, may be chosen freely. One might here observe backward scattering and analyze the energy of the emerging particles. A minor disadvantage of this method may be the uncertainties arising from multiple scattering. In each of the above three methods, thick targets may be used. In the measurements one should observe an extinction of reactions, since the relative reduction in the number of reactions is the fraction χ of particles in the random beam. The minimum value of this fraction is indicated by (2), i.e., of order of $\frac{1}{50}$. The measurements by Bøgh, Davies and Nielsen show indeed a very considerably dip in the number of reactions. It is of great interest to observe the fraction χ of particles in the random beam as a function of penetration depth, varying also other parameters.

Apart from the minimum of the dip, one may study its shape as a function of angle, and the absolute number of reactions. The increase to about normal number of reactions should occur within an angle $\sim C\psi_1$. However, the rise should go above the normal number, and slowly decrease to the normal value for increasing angle. The shape also depends on string vibrations. In these effects it may be noted that the - comparatively smaller - difference in stopping between aligned and random beams gives a trend opposite to the extinction effect, since, for instance, a decrease in electronic stopping permits a nuclear resonance occurring on a larger path length. Whereas this does not affect aligned particles for which no nuclear reactions occur, it may influence the wings of the distribution. It is also clear, partly because of uncertainties in electronic stopping, that unless a large dip can be observed at zero angle of incidence, the results are hardly useful for quantitative studies.

Other experimental methods of interest are studies of distribution in angle of beams having passed through thin monocrystalline foils. Studies of energy loss are also important and of considerable interest, but give less direct information on the behaviour of the beam inside the crystal.

It is seen that the angle ψ_1 in (1) increases with decreasing energy. If it becomes large enough for the particle to hit an atom without encountering its neighbour, the picture clearly needs improvement. In point of fact, for $\psi_1 > a/d$, or $E < E_1 = 2Z_1Z_2e^2d/a^2$, the actual critical angle increases more slowly with decreasing energy than does ψ_1 in (1). We shall not here treat such cases, which comprise the motion of slow heavy ions.

In a detailed study, it is advantageous to introduce the two-dimensional transverse motion in a transverse potential, and apply statistical arguments in the corresponding phase space. Further, it may be mentioned that if one moves away from the direction of strings, but remains close to a lattice plane, there are effects somewhat similar to those of strings, but much weaker and more quickly disappearing. The present results apply for positively charged particles, and for example also for positive mesons; the change of sign of the potential for negatively charged particles implies rather different phenomena, but the same method of approach may be used.

Only a few of the features are sketched in the above discussion. The reader is referred to the detailed paper for a more precise treatment of these questions, as well as those other aspects that were omitted here.

I am much indebted to friends in the theoretical and experimental groups at the institute for illuminating discussions.

* * * * *